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# Measurement of minimum angle of resolution (MAR) in the stereoscopic display using the optotype of stereoscopic stimuli

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**Abstract** — In stereoscopic images, the crossing point of the viewing directions of the two eyes determines the perceived depth. Assuming that accommodation is affected by the positions of the crossing point, the effect of crossing point on minimum angle of resolution (MAR) was investigated. For 40 participants, MAR was measured by two-alternative forced choice where Snellen optotype E of up and down directions were used as two kinds of stimuli. As the crossing point of the viewing direction of the left and right eyes moves farther from the sample display, the ability to identify the direction of letter E decreases at the optotype of the same line thickness. The change of MAR shows linear trends with respect to the optical power change that are the reciprocal of the distance from the participant to the crossing points located out of screen and on screen.

**Keywords** — *minimum angle of resolution (MAR), stereoscopic image, visual acuity, accommodation, convergence.*

DOI # 10.1002/jsid.233

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## 1 Introduction

Since the invention of the stereoscopic by Sir Charles Wheatstone in 1838, many display technologies related to the observation of the stereoscopic images had been reported.<sup>1–3</sup>

In case of the stereoscopic observation of the real object, accommodation and convergence of the two eyes work together to obtain clear stereoscopic vision.<sup>4,5</sup> However, in case of the observation of the stereoscopic images, a phenomenon called the accommodation–convergence conflict had been reported.<sup>6–8</sup> Accommodation is related to the observation of sharp clear image by the change of optical power of crystalline lens inside the eye. If accommodation does not work properly in the observation of the stereoscopic images, the ability to identify the small object or the thin lines is expected to deteriorate. Minimum angle of resolution (MAR) had been used to characterize the identifiable minimum line thickness and line intervals.<sup>9</sup>

Minimum angle of resolution was investigated whether it was affected in the observation of the stereoscopic image. The position of the crossing point between the viewing directions of the left and right eyes determines the perceived depth of the stereoscopic display. At the various positions of the crossing point and the different sizes of the optotype, MAR was measured using two-alternative forced choice (2AFC).<sup>10,11</sup> From the measured result of the correct answer ration, the relation between MAR and the crossing point were analyzed.

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## 2 Theory

Stereopsis occurs as the viewer has the two eyes and the images observed by the left and the right eyes are slightly different. In stereoscopic vision, the left and right eyes of the viewer converge on the object, and accommodation occurs such that the focal length of eyes changes to observe the clear image located at the viewing distance as illustrated in Fig. 1. Change of the focal length of the eye is mostly attributed to the change of the optical power of the crystalline lens inside the eye.

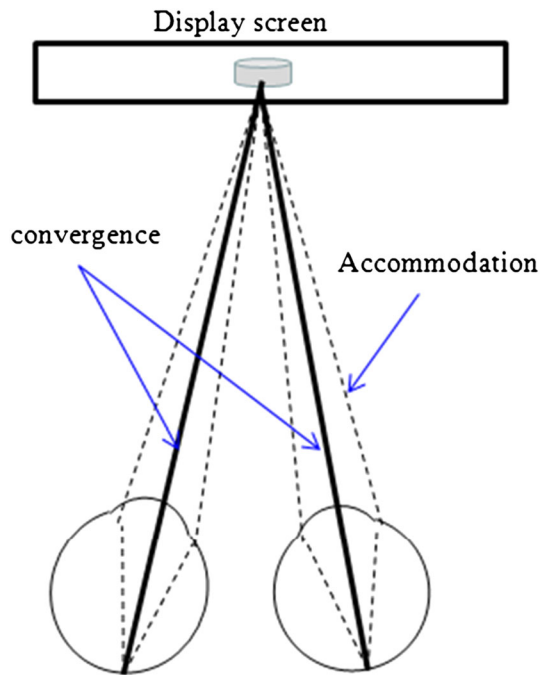
In case of the observation of the stereoscopic images using the stereoscopic display, the image data for the left and the right eyes are not the same. This difference is called the binocular disparity (BD) on the screen. As each eye views the different direction on the screen, the viewing direction of each eye crosses at the position that is not located on the display. Hence, the user perceives 3D object in stereoscopic display to be located at the crossing point as illustrated in Fig. 2. Therefore, the BD on the screen determines the crossing point and deeply related to the perceived depth.

If the user perceives 3D object in the stereoscopic display to be located at the crossing point, the optical power of the eyes tends to change to match the position of the perceived 3D object located in the crossing point. In that case, as the stereoscopic images are actually displayed on the display screen, the images on the stereoscopic display become defocused on the user's eyes as illustrated in Fig. 2(b). Then, the focus of the eyes will shift to the position of the

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Received 10/7/13; accepted 07/30/14.

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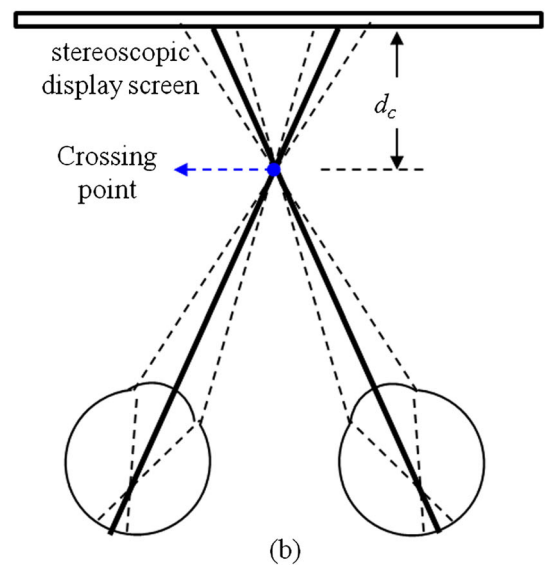
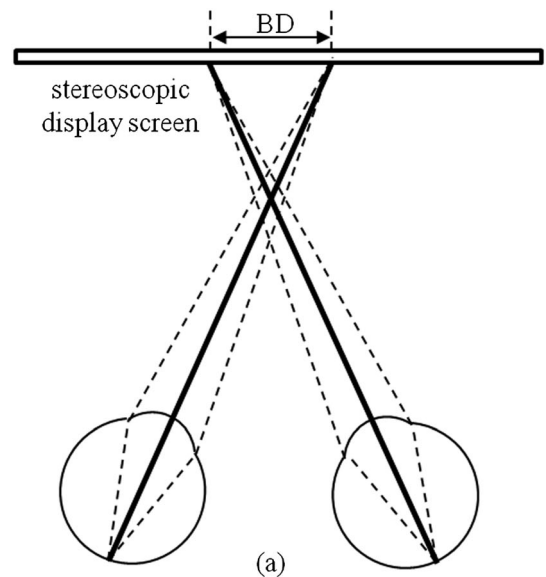


**FIGURE 1** — Accommodation and convergence occurring in the observation of real 3D object or 2D image on the display screen.

stereoscopic display, and the users can observe the clear stereoscopic image, and accommodation of the eye would again change to the crossing points. This phenomenon of the repetitive change of accommodation occurring in the stereoscopic display is known as accommodation–convergence conflict. The existence of such a conflict is one of the major differences between the observation of the real 3D object and 3D object in the stereoscopic display.<sup>6–8</sup> And this is known to cause fatigue and headache in stereoscopic display.

When the focus of accommodation is not on the display screen, the user will observe the defocused image. In this case, the user will have difficulty in resolving the images compared with the condition that the focus of accommodation is on the display screen. Hence, MAR is expected to be worse for the former. If such change of MAR occurs, the distance of the crossing point from the display screen will affect MAR. On the other hand, if the change of MAR does not occur, it means that the focus of accommodation remains on the screen irrespective of the position of the crossing point or the perceived depth of the stereoscopic image.

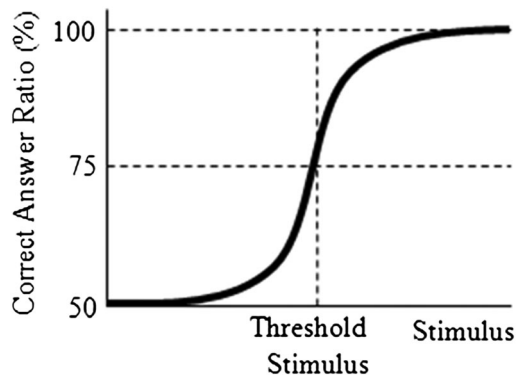
To measure MAR at the given conditions, 2AFC method was employed where two stimuli were given and the user was forced to select one of the two stimuli.<sup>10,11</sup> When the user could identify the two kinds of stimuli, the ratio of the correct answer would asymptotically approach 100%. When the user could not completely discern the stimuli and the user would guess the answer, then the ratio of the correct answer would approach 50%. The typical profile of the psychometric function of the ratio of the correct answer was given as Fig. 3. As half of the 100% and 50%, the correct answer ratio of 75% was generally used as the threshold of the chosen stimulus for 2AFC method.



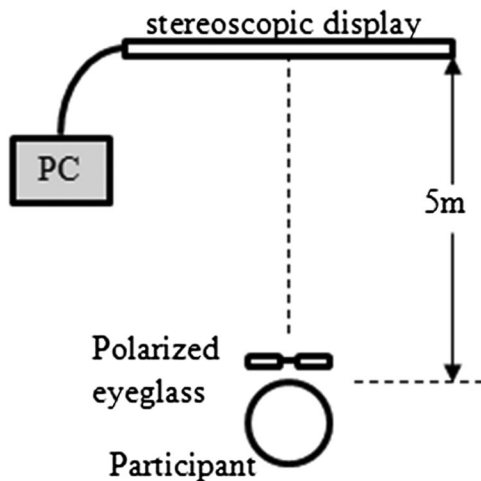
**FIGURE 2** — Accommodation–convergence conflict in the stereoscopic 3D display. Binocular disparity (BD) represents the BD on the screen that is the horizontal difference between the left and the right images of the screen. Crossing point represents the point where the viewing direction of the left and right eyes intersects. (a) Accommodation is at the display screen. (b) Accommodation is at the crossing point.  $d_c$  represents the distance from the display to the position of the crossing point.

### 3 Methods

To characterize the change of MAR with respect to the position to the crossing point in the observation of the stereoscopic display, the following experiment was devised. Figure 4 illustrates the schematic setup of the experiment. Stereoscopic display based on the technology of the patterned retarders and the polarizing eyeglasses was selected as the sample display.<sup>12</sup> Participant was required to wear the polarized eyeglass to perceive the stereoscopic depth. The pixel pitch of the sample display was 0.18 mm, the diagonal size



**FIGURE 3** — Typical profile of psychometric function of two-alternative forced choice method. The horizontal axis represents the stimulus used in the experiment. The vertical axis represents the correct answer ratio.

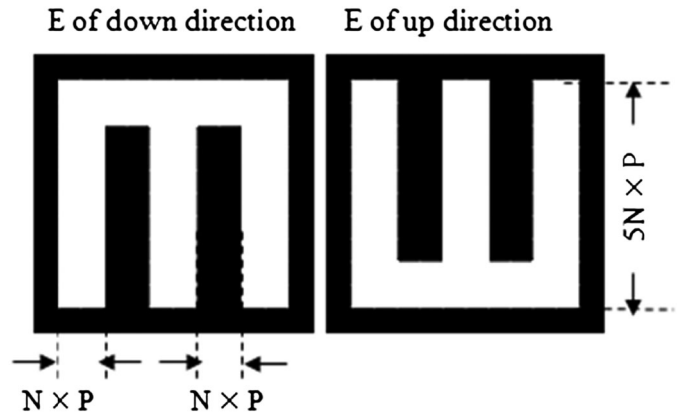


**FIGURE 4** — Schematic layout of the experiment. The viewing distance was 5 m. Participant wore the polarized eyeglass during the experiment.

was 15.6 inch, and pixel numbers of the sample display were  $1920 \times 1080$ .<sup>13</sup> Participant wearing the polarized eyeglass was located on the line normal to the center of the sample display. The distance between the sample display and the participant was selected as 5 m. Input signal of sample display was controlled by Note PC.

A total of 40 participants who had the ability of stereopsis and were in the age range of  $24.6 \pm 2.4$  years were selected. To prevent the effect caused by the difference of the visual acuities of the participants, the visual acuity of all participants were matched to the same visual acuity of 1.0 in decimal scale. For this, the additional correction lens was applied to the participant, if necessary.

Figure 5 illustrates two kinds of optotype used for 2AFC method. E characters of up or down directions were used. Following the rule of Snellen optotype, the line thickness and the distance between the lines were selected to be equal to  $N \times P$ . P was pixel pitch of the sample display. N was integer between 4 and 8. Size of the optotype was selected five times the line thickness. The selected line thickness and the corresponding MAR were listed in Table 1. As optotype on the



**FIGURE 5** — Two kinds of optotype used in two-alternative forced choice method. Pixel pitch, P, of the sample stereoscopic display is 0.18 mm. Line thickness and the distance between the lines are equal to  $N \times P$  where N is integer of 4, 5, 6, 7, and 8. Size of the optotype is  $5 N \times P$ .

**TABLE 1** — Line thickness of optotype and minimum angle of resolution (MAR).

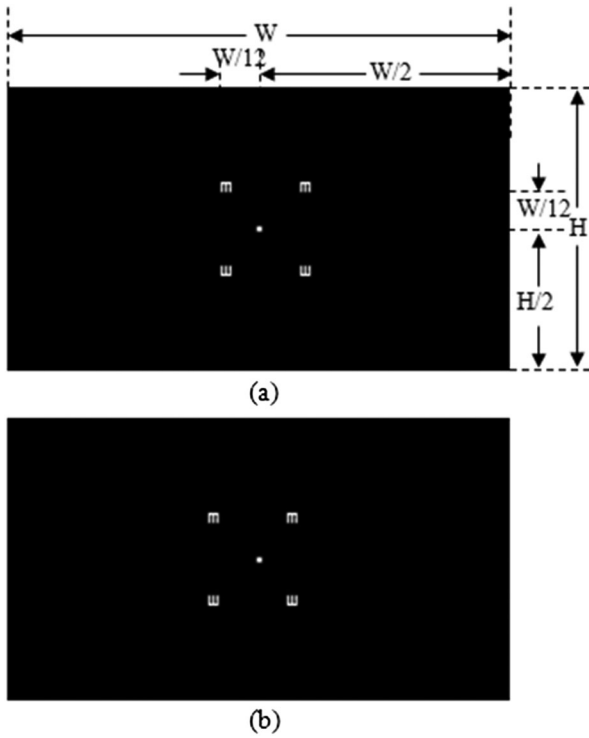
Line thickness of optotype (pixel)	Line thickness of optotype (mm)	MAR (arc minute)
4	0.72	0.50
5	0.9	0.62
6	1.08	0.74
7	1.26	0.87
8	1.44	1.00

Pixel size is 0.18 mm. MAR of 1.0 arc minute corresponds to the ability to identify the line interval of 1.5 mm at the distance of 5 m.

sample display was used for the measurement, the size of the optotype was the multiplication of pixel pitch of the sample display and could be changed discretely because of the finite size of the pixel pitch of the sample displays.

Figure 6 illustrates an example of input signal where four optotype of the same size are used simultaneously. Directions of each of these optotype can be either up or down direction. Positions of these optotype in the input signals for the left and right eyes are horizontally different by the size of BD (BD on the screen). The size of BD determines the position of the crossing point as illustrated in Fig. 2.

Table 2 shows the selected size of BD and the distance of the crossing point from the sample display located at the viewing distance of 5 m. As the pixel pitch of the sample display was 0.18 mm, BD could be represented as the multiplication of the pixel pitch. (+) and (-) signs of the crossing point represent the positions of the crossing point located behind and in front of the display. If the size of BD of the optotype is too large, it will be outside the Panum's range, and the participants will experience the double vision, not the stereoscopic vision. The sizes of BD of the optotype were selected such that the participants were able to observe the stereoscopically fused images.<sup>14-16</sup> Before the main experiment, input signals of BD of -24 pixels (-4.32 mm) and 18 pixels (3.24 mm) were shown to the participant, and each participant was asked whether the participant could observe the



**FIGURE 6** — Input signals for (a) the left eye and (b) the right eye of the stereoscopic display.  $H$  and  $W$  represent the height and width of the stereoscopic display sample, respectively. Four optotypes were used as stereoscopic stimuli. Positions of these optotype for the left and right eyes are horizontally different by the size of binocular disparity on the display screen.

**TABLE 2** — Binocular disparity (BD) on the screen and  $d_c$ , which is the distance from the sample display to the position of the crossing point.

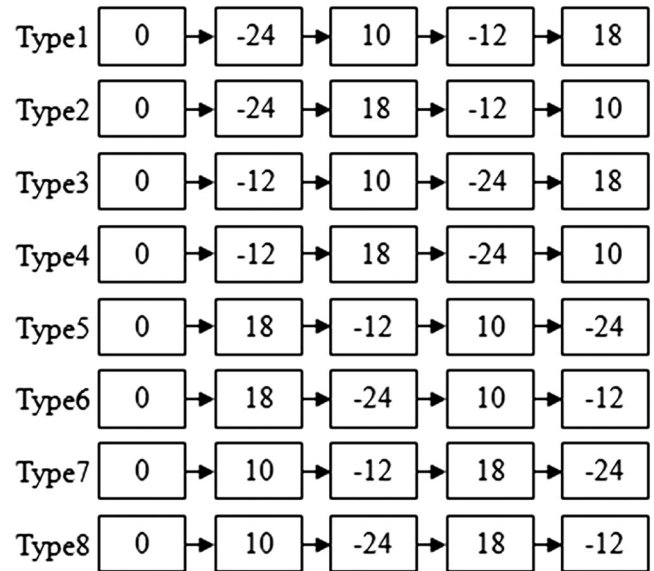
BD (pixels)	BD (mm)	$d_c$ (mm)
-24	-4.32	-311.60
-12	-2.16	-160.81
0	0	0.00
10	1.8	142.41
18	3.24	262.31

(-) and (+) signs represent the position of the crossing point in front of the screen and behind the screen, respectively. The distance from the participant to the sample display is 5 m.

stereoscopically fused images or double images and were instructed about the proper verbal response during the main experiment.

To prevent the effect caused by the experiment sequence, eight kinds of measuring sequence were designed as illustrated in Fig. 7 where the position of the crossing point changed between behind and in front of the screen alternatively. Each of these eight measuring sequences was applied to five participants among 40 participants.

At each conditions of BD, input signals with four optotype of Fig. 6 were shown to the participant for the different optotype size. Direction of each optotype was controlled for each input signal such that the occurrences of up and down directions were randomly distributed but equal in total



**FIGURE 7** — Eight kinds of measuring sequences. Numbers in the box represent the binocular disparity of stereoscopic stimuli.

**TABLE 3** — Result of statistical analysis by paired  $t$ -test of the measured correct answer ratio.

BD (pixels)	Average of the correct answer ratio	Standard deviation of the correct answer ratio	$p$ -value
0	84.39	$\pm 24.6$	—
-12	79.75	$\pm 25.1$	0.203
-24	78.37	$\pm 26.1$	<i>0.039</i>
10	81.25	$\pm 24.7$	0.063
18	79.12	$\pm 26.1$	<i>0.018</i>

$p$ -value less than 0.05 were italicized to notify statistical significant difference.

occurrence throughout the measurement. The participant was asked to tell the direction of each optotype to be up or down.

In the main experiment, five conditions of line thickness and five conditions of BD described in Tables 1 and 2 were used. Hence, a participant observed 25 input signals. Each input signal consisted of four optotype and was shown for less than 8 s. For each participant, the total measurement time was taken less than 5 min. The authors think that the measurement time was short enough not to cause the fatigue. At each combination of the conditions of line thickness and BD, 40 participants answered the direction for four optotype. Hence, the ratio of the correct answers was collected from 160 answers at each condition.

## 4 Results and discussion

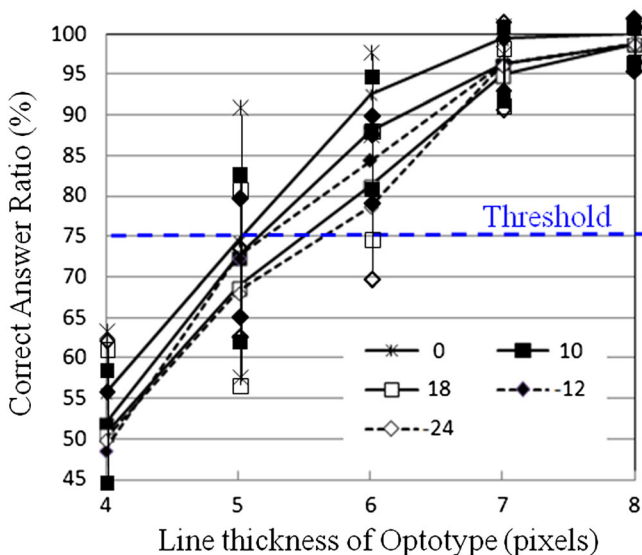
Table 3 shows the average and the standard deviation of the measured correct answer ratio for the different values of BD, where the results at the conditions of the different line thickness were summed together. Paired  $t$ -test of SPSS<sup>TM</sup> was used to compare the result at zero and nonzero BD.<sup>17</sup>

In the case of  $BD = -24$  or  $18$  pixels,  $p$ -values were smaller than  $0.05$ . Hence, the result of the correct answer ratio at  $BD = -24$  or  $18$  pixels were verified to be statistically different from that of  $BD = 0$  pixels. However,  $p$ -values were larger than  $0.05$  in the case of  $BD = -12$  or  $10$  pixels. Hence, the result of the correct answer ratio at  $BD = -12$  or  $10$  pixels were not verified to be statistically different from that of  $BD = 0$  pixels. If MAR is not affected by the BD of optotypes, the results at the different BD should be statistically the same irrespective of the size of BD. The result at  $BD = -24$  or  $18$  pixels was statistically different from the result at  $BD = 0$  pixel, while the result at  $BD = -12$  or  $10$  pixels was not statistically different from the result at  $BD = 0$  pixel. This difference may be explained because the selected conditions of  $BD = -12$  or  $10$  pixels were not large enough to cause significantly different MAR.

The measured ratios of the correct answer with respect to the different line thickness of optotype and the different BD were illustrated in Fig. 8. Standard deviation and the average of the measured ratio were illustrated, too. Each curve of the correct answer ratio showed the tendency of asymptotically approaching  $100\%$  for the thicker lines of the optotype and approaching  $50\%$  for the thinner lines.

The condition of the zero BD corresponded to the case that the crossing point was located on the sample display and there should be no accommodation–convergence conflict. When BD was not zero, the correct answer ratio tended to decrease at the same line thickness. This result means that the participant had difficulty in identifying the optotype of the same size if the crossing point of the optotype was not located on the sample display.

Line thickness corresponding to the threshold of  $75\%$  was calculated from Fig. 8 and converted to MAR for the viewing



**FIGURE 8** — The measured correct answer ratio with respect to the different line thickness of optotype. The horizontal axis represents the line thickness of optotype. The vertical axis represents the correct answer ratio. Numbers on the lower right side represent the size of binocular disparity, represented in pixel numbers.

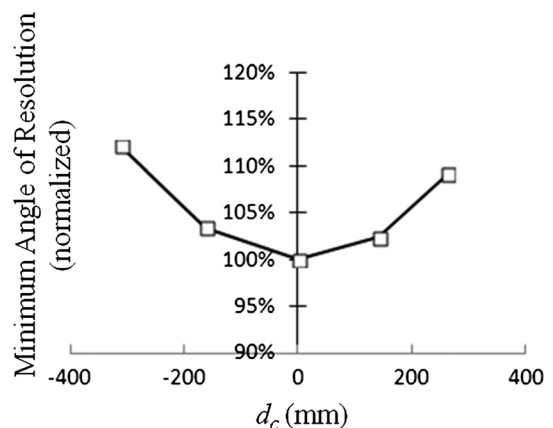
distance of  $5\text{ m}$ . The effect of the positions of the crossing point on MAR is illustrated in Fig. 9 where the vertical axis represents MAR normalized to the value of the zero BD. The horizontal axis represents the distance from the display screen to the crossing point. Figure 9 shows the tendency of increasing MAR for the increasing distance of the crossing point, irrespective of the position of the crossing point behind or in front of the screen.

Optical power of lens is the degree to which a lens converges or diverges light. It is equal to the reciprocal of the focal length measured in meter, and its unit is defined as diopter (D). Figure 10(a) represents the definition of the optic power change. When the crossing point is on the sample display screen at the zero crossing point, the optical power is equal to the reciprocal of the distance from the participant to the sample display and is  $1/5\text{ m} = 0.2\text{ D}$ . When the crossing point is not on the sample, the optic power can be calculated as  $1/(5\text{ m} + d_c)$  where  $d_c$  represents the distance from the display to the position of the crossing point and is listed in Table 2.

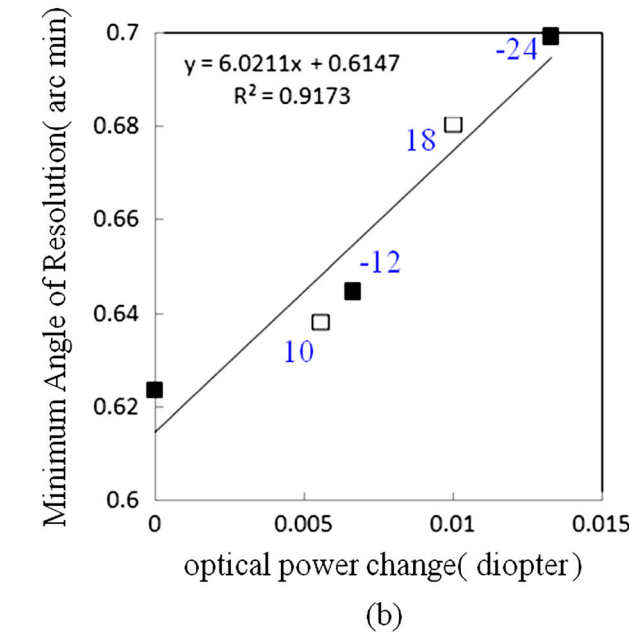
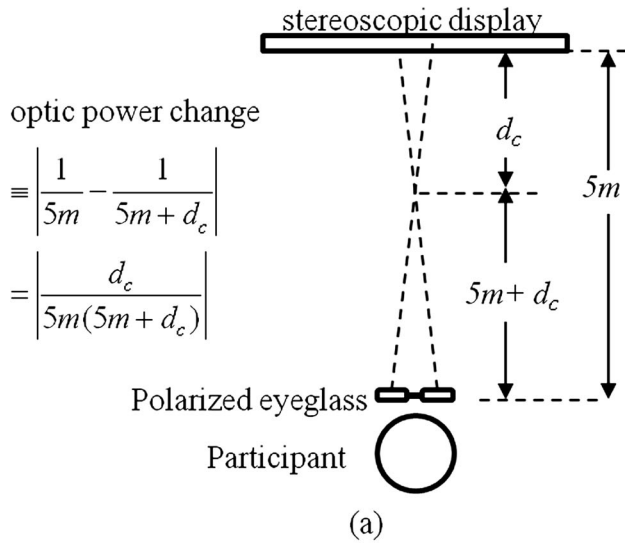
When the position of the viewing object changes, accommodation occurs that the optical power of the eye changes to observe the clear image. In the observation of the stereoscopic display located at  $5\text{ m}$ , the change of the optic powers from the distance of  $5\text{ m}$  to the crossing point of nonzero  $d_c$  can be represented as the optic power change from  $5\text{ m}$  to  $(5\text{ m} + d_c)$ . This relation is represented as Eq. 1.

$$\begin{aligned} \text{Optic power change} &= \left| \frac{1}{5\text{m}} - \frac{1}{5\text{m} + d_c} \right| \\ &= \left| \frac{d_c}{5\text{m}(5\text{m} + d_c)} \right| \end{aligned} \quad (1)$$

When  $d_c$  is much smaller than  $5\text{ m}$ , this optic power change becomes proportional to  $d_c$ . Hence, the change of optic power related to the position of the crossing point increases



**FIGURE 9** — The distance from the display screen to the position of the crossing point versus minimum angle of resolution (MAR). The horizontal axis represents  $d_c$ , which is the distance from the display screen to the crossing point. The vertical axis represents MAR normalized to MAR at  $d_c = 0\text{ mm}$ .



**FIGURE 10** — (a) Definition of optical power change and (b) Optical power change versus minimum angle of resolution (MAR). The horizontal axis represents the optical power change with respect to the position of the sample display. The vertical axis represents MAR. Numbers inside the graph represent the size of binocular disparity, represented in pixel numbers.

as the size  $d_c$  of becomes larger. The optical power changes for the experimental condition of  $d_c$  were illustrated with respect to the measured MAR in Fig. 10(b). MAR of Fig. 10 (b) shows the trends of roughly linear increase as the change of optical power or  $d_c$  increases.

The reason of MAR change may be explained according to the working principle of stereoscopic display of Fig. 2. The change of MAR implies that the eyes do not focus on the position of the sample display. The change of optical power of Fig. 10 should be approximately proportional to the required change of the optical power of the eye if the focus of the eye is assumed to change to observe the clear image of the object located at the crossing point. Although it is not

clear whether the eyes focuses on the crossing point or the intermediate position between the crossing point and sample display, the result shows the focusing position of the eye departs farther from the sample display for the larger distance of the crossing points.

## 5 Conclusion

In the experiment using a stereoscopic display, the resolving ability to identify the direction of optotype of letter E was measured to decrease, and MAR was measured to increase as the crossing point of the viewing direction of the left and right eyes moves farther from the sample display. Change of MAR was approximately proportional to the optic power change, which was approximately proportional to the distance of the crossing point from the sample display.

This result implies that accommodation occurs such that the focus of the eyes shifted from the position of the display screen toward the position of the crossing point.

The result means the viewer cannot discern the less detailed image if 3D objects in the stereoscopic images are far from the sample display. Hence, the result may be considered in the preparation of stereoscopic images such that the focus of the camera may better be on the position of the zero depth, or the detail level of the computer-generated stereoscopic images may be spatially different depending on the information of spatial depth distribution.

## Acknowledgment

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2013R1A1A2005812).

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